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MATHEMATICS FOR SERVICE.*

By Ernest R. von Nardroff.

For a long time mathematics has enjoyed the position of first place in the elementary schools, and in the high schools the time devoted to the subject is nearly as great as that for any other subject. Almost daily for ten years, reaching from the first primary grade up to the conclusion of the second year in the high school, a pupil is required to bend his best energies toward the mastery of mathematics. What are the results? It is needless to state that the average graduate going into business or into any of the learned professions, except engineering, fails to put his mathematics to any use whatsoever. Beyond the requirements of keeping one's accounts and calculating interest and taxes, the ordinary citizen has no need of formal mathematics. The result is that practically everything in this subject learned in the high school is promptly forgotten.

How does it stand with the student of engineering? After taking all the mathematics he can get in the high school he pursues the subject further, either in the pure or applied form, for four years at college. Probably no other subject in an engineering school causes as many failures, and yet I feel confident that not one engineer in ten avails himself of any of the higher mathematics that caused him so much pain and trouble. Nearly everything he does of a mathematical character is done with the aid of formulæ, or by well-settled rules. Anything like an independent mathematical excursion, for which he is miserably prepared, is generally quite out of the question.

Let us finally consider the student of physics. I mean one who aspires to be master of the science, both experimentally and mathematically. If, after his graduation from college, he attempts to take up the reading of advanced mathematical physics, such as the works of Maxwell and Rayleigh as well as many of the articles found in periodicals like the *Philosophical Magazine,* he finds himself baffled by mathematical material quite strange to him. It becomes necessary for him to interrupt and retard his studies in mathematical physics by long detours into the field of pure mathematics. The reason for this is, not that he has not studied a sufficient *quantity* of mathematics, but that he has not studied sufficient *kinds* of mathematics. He needs a little from each of a great many mathematical subjects. In his high school and college courses his time was taken up studying in only a few subjects a mass of material that has no bearing whatsoever upon his work.

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* Read at the meeting of the New York Section.

If, then, class-room mathematics has thus signally failed to render service in the world of realities, some other defence for retaining it to the present extent in our curriculum must be found. There are so many other subjects of a modern character pushing to the front and claiming for themselves the highest service, that, unless mathematics can claim for itself as much as is offered by these other subjects, it will surely be forced to give way. It can, indeed, make a very strong claim, a claim for *culture*. No one who has witnessed a class of eager pupils, vying with each other to be first in finding a solution to some geometrical problem can doubt for a moment the tremendous value of the culture derived from such exercise; and I am glad to say that in recent years the teaching of geometry has been very much improved in the direction of enhancing the culture obtainable from its study. But here, again, mathematics must compete with those modern subjects that also claim high cultural value, and so, in spite of all that can be said and done, mathematics will presently be pushed very hard.

This condition of affairs has been well recognized by representative mathematics teachers, and so several plans have been suggested for making mathematics of value for *service* as well as for culture. " Service " is here employed, not in the indefinite general sense that might include mental growth, but in the sense of direct utility.

One plan that has been suggested is to introduce a large number of technical problems drawn from the trades and allied sources—problems in sheet metal work, brick laying, building construction, railroading, etc. I fear that this plan will not lead to the desired result because to boys and girls not interested in the trades represented, the most important problems would be exceedingly dull. Besides in many cases very lengthy explanations would be required unless the plan happened to be worked out in a school provided with the corresponding shops.

A second plan is to introduce illustrative problems from the field of mechanics and physics, as, for example, problems relating to momentum, kinetic energy, specific gravity, and calorimetry. I cannot approve of this plan because I feel confident that it is pedagogically wrong to illustrate a principle by means of strange material that is harder to understand than the principle being illustrated. It must be remembered that in our high schools physics does not come until the third year.

A third plan, already fairly well worked out, is, though limited in application, somewhat more promising. I refer to the introduction of problems from strictly every-day life. But I should like here to caution against the introduction of methods that are foreign to those appropriate to these problems. For example, in calculating the amount of carpet required to cover a floor one often meets with methods that would lead to results very different from those obtainable from a reputable carpet dealer. The same is true for paper hanging. The proper methods should be learned from practical men. However, the change here suggested, although an important one, is so simple as not to require further comment. Some additional problem material may be obtained by looking into the out-of-school activities and interests of boys and girls.

A fourth suggestion, which I myself would make were the time ripe for it, is to begin physics in the lowest grades. There can be no question of the fact that physics is by far the most comprehensive field for the application of mathematics. Scarcely any subject of mathematics has failed to find some application here, and, further, in many cases the demands of physics have been so far beyond the powers of mathematics to supply, that mathematicians have been either forced then and there to enlarge their own domain or have had it done for them. Should physics, some time in the future, obtain a place in our curriculum as conspicuous as that which mathematics now holds, there would be no lack of material for illustrating the highest mathematics, and there could be no question of the service that mathematics could and must perform in return. But this is a condition not likely to arise in the very near future.

I am, therefore, forced to offer now, for present carrying out, a fifth sugestion. This suggestion is to teach additional mathematical subjects, and in all subjects to emphasize those processes that are useful in scientific and practical fields as well as in every-day life. The illustrative problems should relate to perfectly familiar matters as practical as possible. It will be very desirable in high schools to go far beyond the subjects ordinarily taught. In practical fields a considerable demand is made upon analytical geometry, and calculus is a tool for constant service. It will only be possible to study these subjects by gaining time through the elimination from our present course of all material of little or no practical value, direct or indirect. This elimination is of the greatest importance because, besides the need for time in taking up higher subjects, it is necessary to gain time for more thorough drilling upon such phases of our present subjects as are of first importance in practical work. It will also be necessary to gain time by confining attention to simple problems and forms.

There will undoubtedly be some difference in opinion as to just what topics should be emphasized; which should be given only a small amount of time in passing; and which should be cut out entirely; and indeed I think that the first step taken toward carrying out the plan that I am suggesting should be a thorough investigation into what pure mathematics is really useful in higher mathematical physics, in navigation, in astronomy, in engineering, in the trades, and in ordinary life. In the meantime, for the sake of definiteness, it will not be amiss for me to suggest a few topics that I should like to see emphasized, and a few topics that I should like to see simplified or even eliminated.

In algebra, I would *emphasize* the solution of simple simultaneous equations by the method of substitution. The coefficients employed in the algebraic expressions should not always be simple ones, but occasionally should imitate practical conditions by the introduction of one or two decimal places. The pupils should become skillful in the solution of quadratic equations involving one unknown quantity, either by the use of a formula or by the method of completing the square. The binomial theorem should be much emphasized, especially in the case of simple fractional powers. In this case the first few terms of the infinite series obtained will frequently serve as a suitable approximation in a complicated problem. The graphic solution of the higher quantics and of transcendental equations should be emphasized. It will not only leave the pupil in a position to solve with ease important problems that formerly required a deep study of the theory of equations, but will beside add very much to the cultural value of his study by giving him a distinctly new viewpoint. Further, graphic methods, when once mastered, may easily be remembered for a lifetime.

Much more should be made of methods of approximation, as for example:

$$1.0004 \times 0.9997 \times 1.0002 \div 1.0001 = 1.0002,$$

in which the principle involved is of frequent occurrence in physics.

Another method of approximation is one that I would suggest as a simplification of one commonly employed. The common expression, now current in scientific literature, for two hundred and forty thousand is 24×10^4. I would suggest that we employ the expression 10_4. The space gained in the following example is obvious, as is also the opportunity to perform it mentally with ease.

$$\frac{(2_3 \times 3_3)^2}{2_5} = 18_7.$$

Numerical solutions of algebraic expressions in which numbers are substituted for letters are of very great value. Another point to be emphasized is to solve an equation for each one of its letters in terms of the remaining letters.

In physics we frequently consider a quantity as being in *direct* or *inverse proportion* to some other quantity; for example in the expression,

$$a = \frac{\rotatebox{180}{bc^2}}{\rotatebox{180}{d}},$$

it is important for the pupil to understand that a is directly proportional to c^2 and inversely proportional to d; and conversely, if the pupil is given the facts as to the relationship of *a* to *c* and *d* he should be taught to construct the formula employing *b* as a constant. This is the kind of problem that is constantly arising in elementary physics. We discover experimentally certain relationships between the members of a complex set of quantities, and then seek to express these relationships by a single formula.

Incidentally, I would like to suggest the following method of collecting the principal facts in proportion. It occurred to me in connection with the diagram for Wheatstone's bridge. I express the proportion,

$$\frac{a}{b} = \frac{c}{d},$$

thus:

$$\begin{array}{c|c} a & b \\ \hline c & d \end{array}.$$

The lines, either horizontal or vertical, may be conceived to represent division when passed over mentally. The combination of the vertical and horizontal lines viewed diagonally may be conceived to represent multiplication. If then the two diagonal products are equal, it may be proved that we have a proportion,

(1) When we start in either of two ways with any one of the four letters;

(2) When all the letters are squared;

(3) When the rows are multiplied by the same or different amounts; and

(4) When either row is increased or diminished by any multiple of the other row.

I should also emphasize in connection with the study of algebra the use of logarithms and of the slide rule.

In algebra I would *omit* all factoring with the exception of that of the difference of two squares, and that for a perfect trinomial square. I do not recall any practical use of factoring demanding more than this. Of course Least Common Multiple, Greatest Common Divisor, and the addition of fractions having large polynomials for their denominators could not well be taken up without more factoring than I have indicated, but the processes just indicated are of little or no practical value when they happen to involve elaborate factoring. I ought here to remind you that there is an immense difference between the appearance of a problem designed by a mathematics teacher and that of one arising in every-day life. The coefficients of real problems are rarely simple numbers. They usually have several decimal places and *appear* to be incommensurable. With such numbers many of the methods of factoring ordinarily taught break down completely. I therefore regard the teaching of such trial methods as very pernicious because, while the pupil may very easily solve seemingly difficult problems by them, he is acquiring a false sense of power. What applies to factoring itself applies to all that depends upon factoring.

I would eliminate the study of the square root of large polynomials; of the solution of simultaneous quadratic equations, except by graphic methods; and continued fractions, the practical use of which must be very limited indeed.

I believe much of the difficulty in the study of radicals can be avoided by the omission of the radical sign. Fractional exponents are simpler and accomplish all that is needed.

In geometry there are some propositions that are used thousands of times as frequently as others. I will mention a few of these that occur to me at the moment. They are those that relate to equality of triangles, to the equivalence of triangles, to the similarity of triangles, to parallel lines, to the square of the hypothenuse, to the angle at the circumference, to the relative areas of similar surfaces whether in two or three dimensions, and to relative volumes of similar solids.

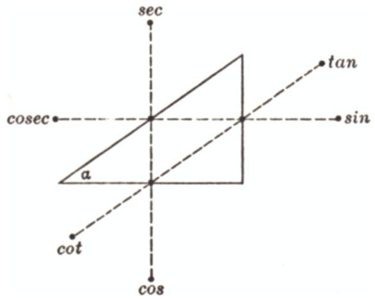
Mensuration should also be emphasized, and in taking up this subject the pupils should be required to calculate from their own measurements the areas of plane diagrams; and similarly with the surfaces and volumes of various solids.

The solution of geometrical problems should not be confined to the use of the compasses and straight edge. The drawing board, with its T square and triangles, should be used also, for these are the instruments employed by those who are solving, in practical fields, the greatest number and variety of geometrical problems.

In geometry I would omit about one third of the second book, and one half of each of the third, fourth and fifth books.

In trigonometry, I would emphasize the solution of right angled and oblique triangles, the summation and difference formulæ for sine and cosine, the relation between the sine and cosine, the use of a full table of trigonometrical formulæ, and the use of a table of the natural and logarithmic values of trigonometric functions.

One of the most valuable devices that I have ever met with for holding in mind trigonometrical relations is the following diagram.



The functions refer to the angle a. Assuming the dots to stand for the functions against which they are placed, or for the lines of the right angled triangle that bear them, the following simple statement will be found to cover an immense system of formulæ. ***Any dot is equal to the product of its two adjacent dots.*** These dots must either be on the same straight line, or else upon an imaginary circumference passing through the functions. As a corollary, any extreme dot is equal to the reciprocal of its opposite extreme dot on the same broken line. Thus:

$$\tan a = \sin a \sec a,$$

$$\tan a = \frac{\sin a}{\cos a},$$

and

$$\tan a = \frac{1}{\cot a}.$$

Or again, the base equals the hypothenuse multiplied by the cosine of the basal angle. I have found this diagram of the greatest use in the higher study of the refraction of light.

I would urge the study of surveying by the use of diagrams of fields, rivers, and mountain peaks. The pupil may select on the diagram appropriate points for taking sights; he may use his ruler in places that in practice would require the steel tape, and a protractor where a transit would ordinarily be used; all to be followed by calculation of results from the data obtained.

Before passing to higher mathematics, I would like to invite attention to the following points in arithmetic: The use of tables of squares, cubes, roots and reciprocals; and a discussion of significant figures and of the customary methods of abbreviated multiplication, division and square root.

In analytical geometry only the simpler theorems relating to conics need be studied, but particular stress should be placed upon the study of the ellipsoid. Much importance should be attached to the study of polar coördinates in three dimensions.

In calculus I would first teach the process of differentiation, illustrating it in a few cases only. I should then provide the pupil with an elaborate reference table of differentials. With integrals I would proceed in the same way. I would endeavor to have the pupil appreciate the significance of differentiation through a contemplation of the variation in purely numerical quantities for which the letters of his equation may be conceived to stand; but I should also make much of the understanding of differentiation in connection with geometrical concepts, with thermal concepts, with pressure concepts, with motion concepts, etc. Also with any physical concept the pupil should understand what is meant by differentiation, not merely in terms of the physical concept itself, but also in terms of its geometrical or algebraic representation. Much is to be made of maxima and minima, a few singularities, involutes and evolutes, definite areas and volumes, and integration between limits and integration by parts. In the case of practical problems too much stress cannot be laid upon the study of the determination of the constant of integration.

The suggestions that I have just made, while they are deliberate in spirit, are rather tentative as regards detail. The only way is for actual teachers of mathematics to make an inquiry into the practical uses of mathematics, and the easiest way for them to make this investigation in higher fields is for them to become students of engineering and mathematical physics. Something in the way of suggestion may be gotten by a glance at Mellor's book on "Higher Mathematics for Students of Chemistry and Physics," published by Longmans & Co. The topics taken up are:

Differential Calculus,
Analytical Geometry,
Singularities of Functions,
Integral Calculus,
Infinite Series,
Solution of Numerical Equations,
Solution of Differential Equations,
Fourier's Theorem,
Theory of Errors and Probabilities,
Calculus of Variations,
Determinants and their Differentiation,
Formulæ,
Reference Tables.

Another book that will prove very suggestive is "Higher Mathematics," by Merriman & Woodward, published by Wiley & Sons. This is a book in which each article has been written by a pure mathematician to meet the needs of scientists. The topics taken up are:

Equations of Higher Degrees,
The Use of Determinants in Calculus,
Projective Geometry,
Hyperbolic Functions,
Harmonic Functions,
Functions of a Complex Variable,
Differential Equations,
Grassman's Space Analysis,
Vector Analysis and Quaternions,
Theory of Errors,
Mathematical History.

The topic of elliptic integrals appears to be omitted. The arrangement in the high school curriculum of the various topics I would recommend to be as follows:

First Year.....	First Term:	Elementary Algebra.
	Second Term:	Plane Geometry.
Second Year...	Third Term:	Advanced Algebra.
	Fourth Term:	Solid Geometry and Mensuration.

Third Year....	Fifth Term:	Trigonometry, Logarithms, Tables of all kinds, Methods of Approximation, the Slide Rule, and Surveying.
	Sixth Term:	Analytical Geometry.
Fourth Year...	Seventh Term:	Differential and Integral Calculus.
	Eighth Term:	Miscellaneous Problems, based as far as possible upon any every-day or scientific material with which the pupil has become familiar toward the end of his course, and presented so as to serve for a general review of his course in mathematics.

The first impression a mathematics teacher is likely to get from this suggestion to introduce analytical geometry and calculus, and to slash up the present traditional courses is that pupils will suffer indigestion from the former and obtain no culture from the latter. Personally, I do not doubt that a pupil in the fourth year who deliberately elects higher mathematics is just the kind of pupil ready to grasp the subject of calculus. As to culture, I feel confident that the useful topics retained may be arranged in a perfectly logical and progressive form. Adding to this the greater breadth involved, it is evident that an increase in culture may be obtained. While, then, mathematics may well for the present continue to be taught principally for culture, we shall not lose, but rather gain ground, by reorganizing our courses with a view to service. Let us strive to have culture and service go hand in hand.

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"Give us, oh give us the man who sings at his work! Be his occupation what it may, he is the equal to any of those who follow the same pursuit in silent sullenness. He will do more in the same time—he will do it better—he will persevere longer. One is scarcely sensible of fatigue whilst he marches to music. The very stars are said to make harmony as they revolve in their spheres. Wondrous is the strength of cheerfulness, altogether past calculation its power of endurance. Efforts to be permanently useful must be uniformly joyous—a spirit all sunshine—graceful from very gladness—beautiful because bright."—THOMAS CARLYLE.